$$u=\frac{1}{96}\left(\frac{(1337+215\sqrt{33})4\pi c^2}{9}\right)^{\frac{1}{3}}[(3298-450\sqrt{33})^{\frac{1}{2}}-2].$$

MECHANICS.

116. **Proposed by C. L. CHILTON, Greensboro, Ala.**

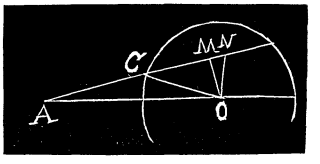
Given, the shaft ABC attached at one end by a pivot to the piston-rod of an engine (at A) and the other to a crank of the wheel CDE (at C). The shaft moves through the distance of two feet in one second from A to B and at the same time turns the crank from C to E. The force propelling the shaft along the constrained course from A to B is 5760 pounds. The mass of the rod and wheel and friction being not considered, what would be the kinetic energy of the machine? or the sum of the moment around O', the center of the wheel?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let AC, the connecting rod$=l$, $OC=r=$one foot, $\angle AOC=\theta$, force 5760 pounds along $AO=P$, component of P along $AC=Q$. Then moment of crank effect about $O=Q.OM$. In the right triangles AOM, AON, $AO:OM=AN:ON$.

$\therefore P:OM=Q:ON$.

$\therefore Q.OM=P.ON$.



Also $ON:OC=\sin OCN:\sin ONC$.

Let $\angle OAC=\varphi$. Then $\angle ONC=\frac{1}{2}\pi-\varphi$, $\angle OCN=\theta+\varphi$.

$$\therefore ON=\frac{r\sin(\theta+\varphi)}{\cos\varphi}=r\sin\theta+\frac{r\cos\theta\sin\varphi}{\cos\varphi}.$$

$$\text{but } \sin\varphi=\frac{r\sin\theta}{l}. \quad \therefore ON=r\sin\theta+\frac{r^2\sin\theta\cos\theta}{\sqrt{(l^2-r^2\sin^2\theta)}}.$$

$$\therefore \text{moment}=Pr\sin\theta\left(1+\frac{r\cos\theta}{\sqrt{(l^2-r^2\sin^2\theta)}}\right).$$

Now θ varies from 0 to π.

$$\therefore \text{Average moment}=\frac{Pr\int_0^\pi \sin\theta\left(1+\frac{r\cos\theta}{\sqrt{(l^2-r^2\sin^2\theta)}}\right)d\theta}{\int_0^\pi d\theta}=\frac{2Pr}{\pi},$$

a result independent of the connecting rod.

$2Pr/\pi=.6366Pr=3666.816$ or 3667 pounds.

117. **Proposed by F. P. MATZ. M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.**

How much lower must *one end* of a heavy uniform chain, wound round the circumference of a perfectly rough vertical wheel, hang than *the other end*, when the chain is on the point of motion?